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Spin waves in superlattices: IV. The exchange-dominated region

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Abstract. Bulk and surface exchange-dominated spin waves in semi-infinite superlattices are considered theoretically within the transfer matrix formalism. The in- and inter-plane exchange constants for two atomic planes at each interface are assumed to differ from appropriate bulk values. The spin-wave spectrum has been calculated numerically for superlattices composed of two different ferromagnetic materials. Considerations are restricted to the case of parallel alignment of the film magnetizations.

1. Introduction

Linear magnetic excitations in layered structures have been analysed theoretically in many papers (Barnaś (1992) and references therein). In most of them the considerations were restricted to the long-wavelength range which is usually accessible in experiments. Comparatively less attention has been paid to the short-wavelength region, where the exchange coupling is dominant and the dipolar interactions are negligible. Several different techniques have been developed for theoretical treatment of the exchange-dominated modes in layered structures, including the Green function method (Dobrzynski *et al* 1986), the transfer matrix formalism (Albuquerque *et al* 1986, Barnaś 1988a, 1992) and the interface rescaling technique (Puszkarski 1988, Puszkarski and Dobrzynski 1989). In this paper we shall apply the transfer matrix method and shall make use of appropriate formulae derived previously (Barnaś 1988a). Within this formalism, one expresses appropriate dispersion equations in terms of the transfer matrix and its eigenvalues. Bulk modes then correspond to eigenvalues of the form $\exp(\pm iQL)$, where Q is real and $-\pi/L \leq Q \leq \pi/L$ with L being the thickness of the elementary unit. The surface modes, on the other hand, can propagate only in those regions where the eigenvalues of the transfer matrix are of the form $\exp(\pm\beta L)$ with $\beta = \kappa$ or $\beta = \kappa + i\pi/L$ (for real κ). In the first paper of this series (Barnaś 1988a) some general dispersion equations for bulk and surface modes in semi-infinite structures were derived for the exchange, magnetostatic and retardation limits. These equations were subsequently applied to the spectrum of magnetostatic modes (Barnaś 1988b) and magnetic polaritons (Barnaś 1990) in ferromagnetic and antiferromagnetic structures. The aim of this paper is to apply them to spin-wave spectrum of semi-infinite superlattices in the exchange-dominated region.

The exchange-dominated part of the spin-wave spectrum of infinite ferromagnetic superlattices was considered by Albuquerque *et al* (1986). The semi-infinite case was discussed by Dobrzynski *et al* (1986), who derived appropriate dispersion equations for

both bulk and surface modes by applying the Green function technique. In both cases the in-plane exchange constants for two atomic planes at each interface were the same as the corresponding bulk values. Here, we consider a more general case, when those constants differ from the bulk values.

The explicit formulae for the relevant transfer matrix are given in section 2. Appropriate dispersion equations for bulk and surface modes are presented in section 3. Some numerical results and final remarks are given in section 4.

2. Transfer matrix

Consider a simple-cubic superlattice with (100) interfaces and with the elementary unit composed of two different ferromagnetic films: one consisting of N_1 and the other of N_2 atomic planes. For simplicity we assume the same lattice parameter a for both materials and restrict the exchange coupling to nearest neighbours. Suppose further that all films are magnetized in the film planes and along an external static magnetic field H_0 . This assumption restricts our considerations to ferromagnetic coupling between the films or to antiferromagnetic coupling but with H_0 strong enough to overcome the inter-layer coupling and to force the parallel alignment of the film magnetizations. Anisotropy fields will be neglected here. We assume additionally that the in-plane exchange parameters for the two atomic planes at each interface differ from the corresponding bulk values. In the following description we shall use the coordinate system with the axis z along the magnetic field (and also along the static magnetization) and with the axis x parallel to the superlattice direction.

It is convenient to introduce dimensionless energy units according to the definition

$$\tilde{E} = E/J^{(1)}S^{(1)}. \quad (1)$$

Consequently, we define \tilde{H}_0 as

$$\tilde{H}_0 = H_0/J^{(1)}S^{(1)}. \quad (2)$$

Following the general formulae derived previously (Barnaś 1988a) one finds the following explicit expressions for the elements of the transfer matrix \mathbf{T} :

$$\begin{aligned} T_{11} = & \{\exp[i(N_1 - 1)k_{\perp}^{(1)}a]/[4\delta^2\epsilon\eta^2 \sin(k_{\perp}^{(1)}a) \sin(k_{\perp}^{(2)}a)]\} \\ & \times \{[\delta(F_{\perp}^{(1)} + \eta F_{\perp}^{(2)}) + F_{\perp}^{(1)}F_{\perp}^{(2)}]^2 \exp[-i(N_2 - 1)k_{\perp}^{(2)}a] \\ & - [\delta(F_{\perp}^{(1)} + \eta F_{\perp}^{(2)}) + F_{\perp}^{(1)}F_{\perp}^{(2)}]^2 \exp[i(N_2 - 1)k_{\perp}^{(2)}a]\} \end{aligned} \quad (3a)$$

$$\begin{aligned} T_{12} = & \{\exp[-i(N_1 + 1)k_{\perp}^{(1)}a]/[4\delta^2\epsilon\eta^2 \sin(k_{\perp}^{(1)}a) \sin(k_{\perp}^{(2)}a)]\} \\ & \times \{[\delta(F_{\perp}^{(1)} + \eta F_{\perp}^{(2)}) + F_{\perp}^{(1)}F_{\perp}^{(2)}][\delta(F_{\perp}^{(1)} + \eta F_{\perp}^{(2)}) + F_{\perp}^{(1)}F_{\perp}^{(2)}] \\ & \times \exp[-i(N_2 - 1)k_{\perp}^{(2)}a] - [\delta(F_{\perp}^{(1)} + \eta F_{\perp}^{(2)}) + F_{\perp}^{(1)}F_{\perp}^{(2)}] \\ & \times [\delta(F_{\perp}^{(1)} + \eta F_{\perp}^{(2)}) + F_{\perp}^{(1)}F_{\perp}^{(2)}] \times \exp[i(N_2 - 1)k_{\perp}^{(2)}a]\} \end{aligned} \quad (3b)$$

$$T_{22} = T_{11}(k_{\perp}^{(1)} \rightarrow -k_{\perp}^{(1)}) \quad (3c)$$

$$T_{21} = T_{12}(k_{\perp}^{(1)} \rightarrow -k_{\perp}^{(1)}) \quad (3d)$$

where

$$\delta = J^{(1,2)}/J^{(1)} \quad (4)$$

$$\epsilon = J^{(2)}/J^{(1)} \quad (5)$$

$$\eta = S^{(2)}/S^{(1)} \quad (6)$$

and $F_{\pm}^{(1)}$ and $F_{\pm}^{(2)}$ are defined as

$$F_{\pm}^{(1)} = 4\Lambda(\varepsilon_{\parallel}^{(1)} - 1) - 1 + \exp(\pm ik_{\perp}^{(1)}a) \quad (7a)$$

$$F_{\pm}^{(2)} = \varepsilon\eta[4\Lambda(\varepsilon_{\parallel}^{(2)} - 1) - 1 + \exp(\pm ik_{\perp}^{(2)}a)] \quad (7b)$$

with

$$\varepsilon_{\parallel}^{(j)} = J_{\parallel}^{(j)}/J^{(j)} \quad (8)$$

for $j = 1, 2$. In the above equations, $S^{(j)}$ and $J^{(j)}$ denote the spin number and bulk exchange constant for the j th material ($j = 1, 2$), $J_{\parallel}^{(j)}$ is the in-plane exchange parameter for an interface atomic plane in the j th material and $J^{(1,2)}$ describes the exchange coupling between the materials. The definitions (7a) and (7b) differ from the corresponding ones introduced previously (Barnaś 1988a) by a factor of $J_1 S_1$. Apart from this, a small change in the notation has been introduced. The parameters $k_{\perp}^{(1)}$ and $k_{\perp}^{(2)}$ are determined by the equations

$$\cos(k_{\perp}^{(1)}a) = 1 + 2\Lambda - \frac{1}{2}(\bar{E} - g^{(1)}\mu_0\mu_B\bar{H}_0) \quad (9a)$$

$$\cos(k_{\perp}^{(2)}a) = 1 + 2\Lambda - (1/2\varepsilon\eta)(\bar{E} - g^{(2)}\mu_0\mu_B\bar{H}_0) \quad (9b)$$

where $g^{(j)}$ is the Landé factor for the j th material, and

$$\Lambda = 1 - \frac{1}{2}[\cos(q_y a) + \cos(q_z a)] \quad (10)$$

with q_y and q_z being the components of the in-plane wavevector q .

The transfer matrix \mathbf{T} is a function of \bar{E} , q_y , q_z , $k_{\perp}^{(1)}$ and $k_{\perp}^{(2)}$. For clarity of notation this dependence has not been written explicitly in the above equations. One can easily prove that the matrix \mathbf{T} fulfils the condition $\det \mathbf{T} = 1$. Apart from this, one finds that \mathbf{T} is invariant under the transformation $k_{\perp}^{(2)} \rightarrow -k_{\perp}^{(2)}$. One can also show that $T_{11}^* = T_{22}$ if $k_{\perp}^{(1)}$ is real, whereas $T_{11} = T_{11}^*$ and $T_{22} = T_{22}^*$ if either $k_{\perp}^{(1)} = i\kappa$ or $k_{\perp}^{(1)} = \pi + i\kappa$, with real κ . Similar relations hold also for T_{12} and T_{21} .

The above formulae for the transfer matrix elements are valid for all \bar{E} except those particular values at which $k_{\perp}^{(1)}$ or $k_{\perp}^{(2)}$ is equal to 0 or $\pm\pi/a$. These special values correspond to the edges of the bulk spin-wave bands in the constituting materials. If $k_{\perp}^{(j)} = 0$ or $k_{\perp}^{(j)} = \pm\pi/a$, the expansion for spin-wave amplitudes, which was used when deriving equations (3a)–(3d) (Barnaś 1988a), is not applicable. In that case, however, there is another particular solution of the appropriate difference equation and one can use the following expansion:

$$S_n^{\pm}(q)|0\rangle = A_{m,j}^{\pm}(\pm 1)^{n-(m-1)N} + A_{m,j}^{-}[n - (m-1)N](\pm 1)^{n-(m-1)N} \quad (11)$$

where $n(m)$ is the atomic plane (elementary unit) index, $A_{m,j}^{\pm}$ are constants and the upper (lower) sign corresponds to $k_{\perp}^{(j)} = 0$ ($k_{\perp}^{(j)} = \pm\pi/a$). Applying equation (11) together with appropriate boundary equations, one can derive the transfer matrix corresponding to the special values of \bar{E} . Its explicit form, however, will not be given here.

3. Dispersion equations for bulk and surface modes

The spectrum of bulk modes in superlattices under consideration is determined by the dispersion equation

$$\cos(QL) = \frac{1}{2}(T_{11} + T_{22}) \quad (12)$$

where $L = (N_1 + N_2)a$ and Q is the wavevector component along the superlattice direction. For all \vec{E} , except the particular values discussed above, the transfer matrix elements are given by equations (3a)–(3d). The special cases have to be considered separately, as discussed in section 2.

To find surface modes propagating in semi-infinite structures we shall apply the general formulae derived in a recent paper (Barnaś 1988a). Following them one can write the dispersion equation for the surface modes in the form

$$D_1 D_2 (T_{11} - T_{22}) - D_1^2 T_{21} + D_2^2 T_{12} = 0 \quad (13)$$

where D_1 and D_2 are given by the expressions

$$D_{1(2)} = \mp 1 \pm [1 - 4\Lambda(\epsilon_{\parallel}^s - 1)] \exp(\mp ik_{\parallel}^{(1)}a). \quad (14)$$

The upper (lower) sign in the above expression corresponds to D_1 (D_2) and the parameter ϵ_{\parallel}^s is defined as

$$\epsilon_{\parallel}^s = J_{\parallel}^s / J^{(1)} \quad (15)$$

where J_{\parallel}^s is the exchange integral between spins in the surface atomic plane, which usually differs from the corresponding value inside the structure. We have also assumed that the material corresponding to $j = 1$ is at the surface of the semi-infinite structure.

The decay parameter β_s which corresponds to a particular solution $\vec{E} = \vec{E}_s$ of the dispersion equation (13) can be calculated from one of the formulae

$$\exp(-\beta_s L) = T_{11} + T_{12}(D_2/D_1)|_{\vec{E}=\vec{E}_s}, \quad (16a)$$

or

$$\exp(-\beta_s L) = T_{22} + T_{21}(D_1/D_2)|_{\vec{E}=\vec{E}_s}. \quad (16b)$$

The above expressions are equivalent in a general case. If, however, $D_1 = 0$ at $\vec{E} = \vec{E}_s$, then equation (16b) is applicable. If $D_2 = 0$ at $\vec{E} = \vec{E}_s$, one can make use of equation (16a). Only those solutions for which

$$|\exp(-\beta_s L)| < 1 \quad (17)$$

describe surface modes. One may say equivalently, that only solutions for which the condition $\text{Re}(\beta_s) > 0$ is fulfilled correspond to surface modes. (There is an error in previous papers (Barnaś 1988a, b), where the condition $\beta_s > 0$ should read $\text{Re}(\beta_s) > 0$.) If $\text{Im}(\beta_s) = 0$, the solutions are of the *acoustic* type whereas, for $\text{Im}(\beta_s) = \pi/L$, the modes are of the *optic* type. In a general case a surface mode of a superlattice is of the

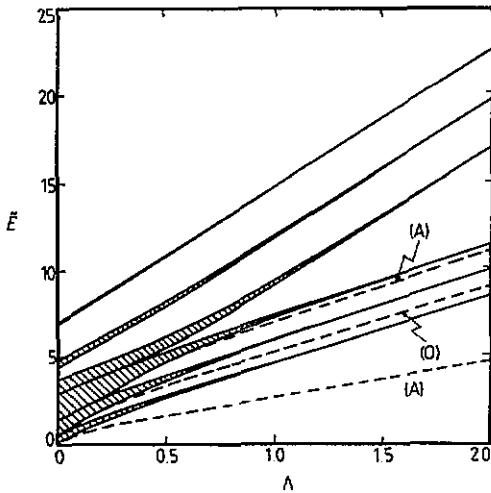


Figure 1. Spectra of bulk (shaded regions corresponding to $-\pi/L \leq Q \leq \pi/L$) and surface (broken curves) exchange modes for $\eta = 1$, $\epsilon = 2$, $\delta = 1.4$, $H_0 = 0$, $\epsilon_{\parallel}^{(1)} = \epsilon_{\parallel}^{(2)} = 1$, and $\epsilon_{\perp}^s = 0.5$.

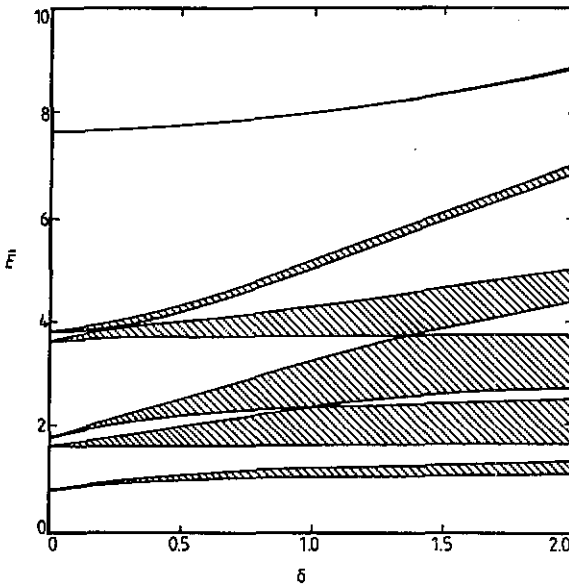


Figure 2. Spectra of bulk modes versus the inter-layer exchange parameter δ for $\eta = 1$, $\epsilon = 2$, $\Lambda = 0.2$, $H = 0$ and $\epsilon_{\parallel}^{(1)} = \epsilon_{\parallel}^{(2)} = 1$.

acoustic type if equivalent magnetic moments in adjacent elementary units precess in phase. If they precess in anti-phase, the modes are of the optic type.

4. Numerical results

The dispersion equations (12) and (13) for bulk and surface spin-wave modes are convenient for numerical calculations. As an example we shall apply them to superlattices with the elementary unit consisting of $N_1 = 3$ atomic planes of one material

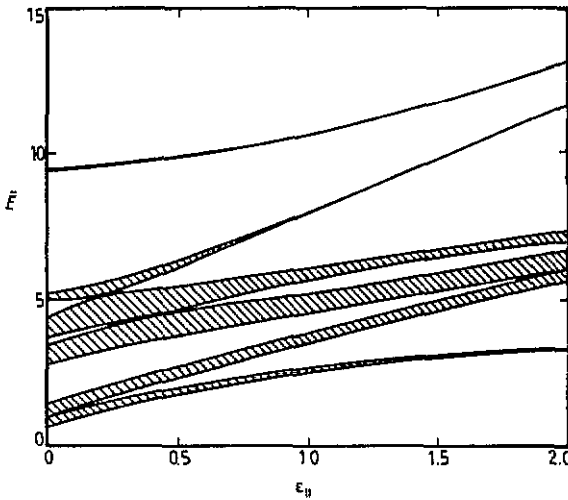


Figure 3. Dependence of the spectra of bulk modes on the exchange parameter ϵ_{\parallel} at the interfaces: $\epsilon_{\parallel} = \epsilon_{\parallel}^{(1)} = \epsilon_{\parallel}^{(2)}$. The other parameters assumed here are $\eta = 1$, $\epsilon = 2$, $\delta = 1.4$, $\Lambda = 0.5$ and $H_0 = 0$.

($j = 1$) and $N_2 = 3$ atomic planes of the other one ($j = 2$). The corresponding spin-wave spectrum is shown in figure 1, where the shaded regions limited by the full curves describe the subbands of bulk modes and the broken lines correspond to the surface modes, *optic* (O) and *acoustic* (A) as indicated. The spin-wave energy is plotted against the parameter Λ . As one could expect, there are six subbands of the bulk modes. (The uppermost band is very narrow for the parameters assumed in figure 1.) For a ‘weakened’ exchange coupling at the surface—as assumed in figure 1—one finds additionally two different surface branches of the *acoustic* type and one of the *optic* type.

In figure 2 the spectrum of bulk modes is shown as a function of the parameter δ , i.e. as a function of the exchange coupling between the two materials. For $\delta = 0$ ($J^{(1,2)} = 0$) the structure is equivalent to a stack of decoupled layers. Consequently, the modes propagating in different films are also decoupled and each state of the stack is highly degenerate. The degeneracy, however, is lifted by a non-zero inter-layer coupling and a characteristic band structure develops from discrete energy levels.

It is also interesting to analyse the influence of the in-plane exchange coupling $J_{\parallel}^{(j)}$ at the interfaces on the spin-wave frequency. For bulk modes this dependence is shown in figure 3, where $\epsilon_{\parallel}^{(1)} = \epsilon_{\parallel}^{(2)} \equiv \epsilon_{\parallel}$ is assumed. As one can see, all bands shift upwards with increasing ϵ_{\parallel} .

The method presented here is based on the transfer matrix T derived in the complex quasi-momentum representation. Another possibility is to use the transfer matrix in the ‘site’ (atomic plane) representation (Barnaś 1992). Both methods give equivalent spin-wave spectra, as they should do.

There are several techniques which can be used for experimental investigations of the spin-wave modes in layered structures. The most suitable are the Brillouin light scattering and spin-wave or ferromagnetic resonances. In some experimental configurations the dipolar coupling is negligible and the corresponding spectra can be interpreted within the approximation described above.

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References

- Albuquerque E L, Fulco P, Sarmento E F and Tilley D R 1986 *Solid State Commun.* **58** 41
Barnaś J 1988a *J. Phys. C: Solid State Phys.* **21** 1021
— 1988b *J. Phys. C: Solid State Phys.* **21** 4097
— 1990 *J. Phys.: Condens. Matter* **2** 7173
— 1992 *Phys. Rev. B* at press
Dobrzynski L, Djafari-Rouhani B and Puzkarski H 1986 *Phys. Rev. B* **33** 3251
Puzkarski H 1988 *Acta Phys. Pol. A* **74** 701
Puzkarski H and Dobrzynski L 1989 *Phys. Rev. B* **39** 1819